Corrections to

Conditional Monte Carlo: Gradient Estimation and Optimization Applications
(Kluwer International Series in Engineering and Computer Science, 392)
by Michael C. Fu and Jian-Qiang Hu, published 1997
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- p.13, Theorem 1.2: In the statement of the theorem, $\tilde{\Theta}$ should be a compact set, and $\tilde{D}$ should be a set of countably many points. The sentence immediately following the theorem should be modified as follows: "automatically satisfies" should be "implies that in order to satisfy"; and append to the end, "one simply needs to show that $\sup_{\theta\in\tilde{\Theta}}\left|\frac{dL}{d\theta}\right|$ is integrable."
- p.15: negative sign missing twice on lines 6 and 16; should be $-\frac{1}{e\theta}$.
- p.45, D/M/1 queue: θ is a "scale" parameter, not a "location" parameter, so should have $dX/d\theta = X/\theta$ instead of $dX/d\theta = 1$, but conclusion $d^2X/d\theta^2 = 0$ still holds.

  Missing term of λ should multiply expression for $\frac{d^2E[T]}{d\theta^2}$, i.e.,

  $$\frac{d^2E[T]}{d\theta^2} = \lambda\left(\frac{2}{(1-\sigma)^2}\frac{d\sigma}{d\rho} + \frac{2\rho}{(1-\sigma)^3}(\frac{d\sigma}{d\rho})^2 + \frac{d^2\sigma}{d\rho^2}\frac{\rho}{(1-\sigma)^2}\right),$$

  Definition of A_n^* should be mod a, i.e.,

  $$\begin{aligned} A_n^* &= \text{residual interarrival time when } C_n \text{ enters service} \\ &= a - [(T_n - X_n) \bmod a] = a - [(T_{n-1} - a) \bmod a] \\ &= a - [(X_{n-1} - A_{n-1}^*) \bmod a], \end{aligned}$$

  where mod a is the *remainder* after dividing by a.
- p.46, last four lines, three occurrences: X_{n-1} should be ΔX_{n-1}.
- p.47: term "$\frac{d^2X_n}{d\theta}$ +" missing before left brace on RHS of equation (2.16), i.e., should read

  $$\frac{d^2T_n}{d\theta} = \frac{d^2X_n}{d\theta} + \left\{ \right.$$

  For (2.17), assume convention $\frac{d^2T_{(m,0)}}{d\theta^2} = 0$.
- p.63, l.10: $\cup$ should be $\cap$.
- p.64, first line: missing "$|z_k$", i.e., should read

  $$E\left[\sum_{k\in\Gamma(n)}\lim_{\Delta q\to 0}\frac{E[(V_i(q+\Delta q) - V_i(q))\mathbf{1}(\mathcal{B}_k)|z_k]}{\Delta q}\right]$$
- p.65, Section 2.3.2 coupling construction:

  $$\tilde{D}_i = D_i,\ i = 1, ..., k-1,\ \tilde{D}_k = \xi_k^-,\ \tilde{D}_{k+1} \sim F,\ \tilde{D}_i = D_{i-1},\ i = k+2, ...$$
- p.68, RHS of part (b) in Lemma 2.5: $\lim_{\Delta q\to 0}$ missing.

- p.69, ll.3-4 (two occurrences): "$\sup_q$" should be "$\sup_{\Delta q}$".
- p.178, RHS of equation (4.59) and preceding equation (three lines above): $p(s'; s; \theta)$ should be removed.
- p.186: it is the right-hand derivative being considered (not specified)
  l.5: $\mathbf{1}\{X_{n+1} > t - X_n\}$ should be $\mathbf{1}\{X_{n+1} > t - T_n\}$
  ll.8-9: should read "DNP corresponds to $T_{n+1} = t^+$ and PP corresponds to $T_{n+1} = t^-$."
- p.192, Example 5.5: expectation should be $E[Y] = 0.5(\theta^2 - \theta + 1)$.
- p.297: missing "+" in expression for cost function $J(s, q)$, i.e., should read

$$J(s,q) = cE[D] + \frac{K + h(s - E[D] + \lambda q(s + \frac{q}{2})) + (h+p)E[D]e^{-\lambda s}}{1 + \lambda q},$$

- p.333, ll.1-3: all three lower integrand limits z^* should be $\sigma\sqrt{T} - z^*$;
  l.5: $Z > z^*$ should be $Z > \sigma\sqrt{T} - z^*$;
  ll.7,9,11: $-z^*$ should be $z^* - \sigma\sqrt{T}$.

  Or, replacing z^* with d_1, and introducing d_2, i.e.,

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\frac{S_0}{K} + (r + \sigma^2/2)T\right), \quad d_2 = d_1 - \sigma\sqrt{T},$$

  the equations should read as follows:

$$\begin{aligned}
E[L(S_0, K; r, \sigma)] &= S_0 N(d_1) - K e^{-rT} N(d_2), \\
\int_{-d_2}^{\infty} dN(z) &= N(d_2), \\
\int_{-d_2}^{\infty} S_T dN(z) &= S_0 e^{rT} N(-d_2), \\
\int_{-d_2}^{\infty} z dN(z) &= N'(d_1), \\
\int_{-d_2}^{\infty} z S_T dN(z) &= \sigma\sqrt{T} S_0 e^{rT} N(-d_2) + S_0 e^{rT} N'(-d_2),
\end{aligned}$$

  where $N(\cdot)$ denotes the standard normal c.d.f. Also note that $S_T > K$ is equivalent to $Z > -d_2$, therefore,

$$\begin{aligned}
E\left[\frac{\partial L}{\partial K}\mathbf{1}\{S_T > K\}\right] &= -e^{-rT} N(d_2), \\
E\left[\frac{\partial L}{\partial S_0}\mathbf{1}\{S_T > K\}\right] &= N(-d_2), \\
E\left[\frac{\partial L}{\partial r}\mathbf{1}\{S_T > K\}\right] &= K T e^{-rT} N(d_2), \\
E\left[\frac{\partial L}{\partial \sigma}\mathbf{1}\{S_T > K\}\right] &= S_0\sqrt{T} N'(-d_2), \\
E\left[\frac{\partial L}{\partial T}\mathbf{1}\{S_T > K\}\right] &= \frac{\sigma S_0}{2\sqrt{T}} N'(-d_2) + K r e^{-rT} N(d_2),
\end{aligned}$$

- p.335, line just above Example 8.4: $h(X_N; S_{t_{n-1}})$ should be $h(X_N; S_{t_{N-1}})$.
  In Example 8.5, $S_{t_{N-1}}$ should be $S_{t_{n-1}}$.

- p.336, line just above Example 8.5: should be

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2 T/N}} e^{\frac{-(\ln x-(r-\sigma^2/2)T/N)^2}{\sigma^2 T/N}}.$$

- p.339: (8.11) is the estimator for $\partial E[L]/\partial\theta$, and (8.12) is the estimator for $\partial E[\tilde{L}]/\partial\theta$.
- p.339, l.-2 (twice); p.340, ll.2-3; L should be $\tilde{L}$.
- p.344, l.3: $\sqrt{t}$ should be $\sqrt{t-in}$.
- p.346: The general estimator for $dE[\tilde{L}]/d\theta$ should read

$$\sum_{i=1}^{\eta(T)} \mathbf{1}\left\{\bigcap_{j=1}^{i-1} S_{t_j^-} \leq s_j\right\} \frac{\partial h^{-1}(y_i^*)}{\partial\theta} f_i(h^{-1}(y_i^*)) \left\{E\left[\tilde{L}\left|\bigcap_{j=1}^{i-1} S_{t_j^-} \leq s_j, S_{t_i^-} = s_i^-\right.\right] - (s_i-K)e^{r(T-t_i)}\right\}$$

$$+\sum_{i=1}^{\eta(T)} \mathbf{1}\left\{\bigcap_{j=1}^{i-1} S_{t_j^-} \leq s_j, S_{t_i^-} > s_i\right\} \frac{\partial}{\partial\theta}\left[(S_{t_i^-}-K)e^{r(T-t_i)}\right]$$
$$+\mathbf{1}\{S_{t_1^-} \leq s_1, ..., S_{t_{\eta(T)}^-} \leq s_{\eta(T)}\} \frac{\partial}{\partial\theta}\left[(S_T-K)^+\right],$$

where $$y_i^* = (s_i - \sum_{j=i}^{\eta(T)} D_j \exp\left(-r\sum_{k=i+1}^{j} \tau_k\right); \tilde{S}_{t_{i-1}}, \tau_i), \quad i=1,...,\eta(T).$$

An estimate of $E\left[\tilde{L}\left|\bigcap_{j=1}^{i-1} S_{t_j^-} \leq s_j, S_{t_i^-} = s_i^-\right.\right]$ can be obtained by simulating to estimate the call payoff, starting at time t_i with stock price$S_{t_i^+} = s_i - D_i$ and remaining thresholds $s_{i+1},\ldots,s_{\eta(T)}$.

- p.351, 2nd paragraph: " ..., will always be 0" insert at end "except when $\theta = h$".
- p.352: right-hand estimator for $\Delta h > 0$ is missing the IPA portion of the estimator:

$$\frac{\partial L}{\partial h} = \frac{L}{h},$$

i.e., should read

$$\begin{aligned}\frac{\partial E[L]}{\partial h} &= E\left[\frac{\partial L}{\partial h} + \lim_{\Delta h\to 0^+} \frac{P(\eta(h+\Delta h)=\eta(h)-1|\eta(h))}{\Delta h}\left[L^{PP}-L^{DNP}\right]\right] \\ &= E\left[\frac{L}{h}\right] + E\left[\frac{\eta\cdot f(\eta h)}{F((\eta+1)h)-F(\eta h)}\right] E\left[L^{PP}-L\right].\end{aligned}$$

- p.353, l.4: missing closing parenthesis ")".
- p.357, l.5: missing final closing parenthesis ")".